9/12/16

DOON UNIVERSITY, DEHRADUN

End Sem Examination, First Semester, 2016

School of Physical Sciences

Mathematics Elective Course

Course: MAG-102: Finite Element Method

Time Allowed: 2 Hours **Maximum Marks: 100**

Attempt all questions from section A, any four questions from section B and any two questions from section C.

Section: A (4×5 = 20 Marks)

(1) Write the finite difference equivalent of the following ODE:

$$y'' + y + 1 = 0.$$

(2) Use Bender-Schmidt formula to solve the heat conduction problem

$$\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$$

with the conditions $u(x,0) = 4x - x^2$ and $u(0,t) = u(4,t) = 0$ taking $h = 1$.

(3) Write the possible form of the trial functions for the BVP

$$y'' + e^x y = x^2,\ y(0) = 1,\ y(1) = 2.$$

(4) Obtain the residual for the boundary value problem

$$y'' - xy' + y = 0,\ y(0) = 0,\ y(1) = 4,$$

where the approximate solution is $y(x) = 4x + C_1 x(x-1) + C_2 x^2(x-1)$.

Section: B (10 × 4 = 40 Marks)

(1) (a) From the following table, find the value of $e^{1.4}$ using suitable interpolation formula

x	1.0	1.2	1.5	1.6	1.9
e^x	2.71828	3.32012	4.48169	4.95303	6.68589

(b) Find the fundamental functional, $G(x, y, y')$ used in Reyleigh-Ritz method for the BVP:

$$y'' - e^x y = x^2; y(a) = A, y(b) = B$$

and then find the variational $I[y(x)]$.

(2) Solve the system

$$6x + y + z = 20$$
$$x + 4y - z = 6$$
$$x - y + 5z = 7$$

using Gauss-Seidel method.

(3) For the BVP: $y'' - 16y = 0, y(0) = 0, y(1) = 100$ the approximate solution is assumed as

$$y(x) = 100x + C_1 x(x-1) + C_2 x^2(x-1).$$

Compute the coefficients by Reyleigh-Ritz method without FEM i.e. applied to whole domain $[0, 1]$.

(4) Consider the BVP

$$y'' + 2y' + y = e^x; y(0) = 2, y(1) = 6.$$

By taking $h = 1/2$ write the node equation, where the weight function to be multiplied is $\phi_1(x)$.

(5) For the BVP in (4) write the expressions for $\frac{\partial I}{\partial C} = 0$ in case the problem is solved by Reyleigh-Ritz method with $h = 1/2$.

Section: C

$(20 \times 2 = 40$ Marks)

(1) Apply collocation method to solve the BVP

$$y'' - 16y = 0, y(0) = 10, y(1) = 10$$

by assuming the approximate solution

$$y = 10x + C_1 x(x-1) + C_2 x^2(x-1).$$

(2) Solve the BVP

$$y'' - y = x,\ y(0) = 2,\ y(1) = 4$$

by Finite Element Method dividing the domain into two equal elements and choosing the basis functions as linear polynomials by Galerkin's approach.

(3) Apply Reyleigh-Ritz method to solve the BVP

$$y'' + y - x^2 = 0,\ y(0) = 0,\ y(1) = 2$$

for $h = 1/2$.