12/12/16

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2016

School of Physical Sciences

M.Sc.(Mathematics)

Course: MAC-401: Finite Field

Time Allowed: 3 Hours | Maximum Marks: 100

Note:

1. Attempt All Questions from Sections A.
2. Attempt Any seven Questions from Sections B.
3. Attempt Any three Questions from Sections C.

SECTION: A

($10 \times 2 = 20$ Marks)

1. The minimal polynomial for $\sqrt[3]{2}$ over Q is .... and is of degree .... over Q.
2. If $Q(\xi_p)$ is cyclotomic field of p^{th} roots of unity, where p is prime, then $[Q(\xi_p) : Q] =$ ....
3. Define seprable polynomial over a field F.
4. Let F be a field of characterstic p. Then for any $a, b \in F$. We have $(a+b)^p =$ ..... and $(ab)^p =$ .....
5. Define characteristic of a field.
6. Define multiplicity of an element a of field K.
7. Let C be the field of complex numbers is algebric over R, then $[C : R] =$ and .... are basis of C over R.
8. Let K_1 and K_2 be two finite extensions of a field F contained in K. Then $[K_1K_2 : F]$.....$[K_1 : F][K_2 : F]$.
9. A generator of a cyclic group of all n^{th} roots of unity is called ....
10. An element $a \in F_p$ is a primitive n^{th} roots of unity if and only if ....

SECTION: B

(7 × 5 = 35 Marks)

1. Show that the algebric closure of a countable field is countable.
2. Show that a finite normal extension is a minimal splitting field of some polynomial.
3. Show that a finite extension is algebric, however the converse is not true.
4. Show that $x^{p^n} - x$ is the product of monic irreducible polynomials in $F_p[x]$ of degree d,d dividing n.
5. Prove that $sinm^0$ is an algebric number for every integer m.
6. Find a basis of $Q(\sqrt{3}, \sqrt{5})$ over Q.
7. Prove that 60^0 is constructable.
8. Find the smallest splitting field of $x^4 + 2$ over Q.Also find its degree over Q.

SECTION: C

(15 × 3 = 45 Marks)

1. Let K be a finite extension of F and L, a finite extension of K. Then show that L is a finite extension of F and [L:F] = [L:K][K:F].
2. Let $a \in K$ be algebric over F.Then show that
(a) there exist a unique monic irreducible polynomial $p(x) \in F[x]$ such that $p(a) = 0$,
(b)if there exist a nonzero polynomial $q(x) \in F[x]$ such that $q(a) = 0$ then $p(x)$ divides $q(x)$.
3. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Then there exist an extension K of F such that $[K : F] \leq n!$ and K has n roots of $f(x)$.
4. Let F be a field of characteristic p. Then show that for any polynomial $f(x) \in F[x]$, $f' = 0$ iff $f(x) = g(x^p)$ for some polynomial $g(x) \in F[x]$.