19/12-16

Roll No. | | | | | | | | | | | | |

Int. M.Sc.-II END SEMESTER EXAMINATION 2016-17
Department of Mathematics, SOPS,
Doon University Dehradun
Partial Differential Equation & System of ODE (MAC-107)
Time: 03 Hour **Total Marks: 100**

Note: Attempt ALL the questions.

Section A

Attempt ALL: **(2x10=20)**

1. The integral surface for the Cauchy problem $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$ which passes through the circle $z=0, x^2+y^2=1$ is:
   (a) $x^2+y^2+z^2+2zx-2yz-1=0$
   (b) $x^2+y^2+z^2+2zx+2yz-1=0$
   (c) $x^2+y^2+z^2-2zx-2yz-1=0$
   (d) $x^2+y^2+z^2+2zx+2yz+1=0$
2. The vertical displacement $u(x,t)$ of an infinitely long elastic string is governed by the initial value problem $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}, -\infty<x<\infty, t>0, u(x,0)=-x$ and $\frac{\partial u}{\partial t}(x,0)=0$, the value of $u(x,t)$ at $x=2$ and $t=2$ is equal to:
   (a) 2, (b) 4, (c) -2, (d) -4
3. The general solution of the partial differential equation $\frac{\partial^2 z}{\partial x\partial y}=x+y$ is of the form: (a) $\frac{1}{2}xy(x+y)+F(x)+G(y)$ (b) $\frac{1}{2}xy(x-y)+F(x)+G(y)$
   (c) $\frac{1}{2}xy(x-y)+F(x).G(y)$ (d) $\frac{1}{2}xy(x+y)+F(x)+G(y)$
4. The Complementary function of $(D^2-4DD'+4D'^2)z=(x+y)$ is?
5. The general solution of a linear first order equation $Pp+Qq=R$ is given by the equation________
6. The PDE of all spheres whose centres lie on z-axis and given by the equation $x^2+y^2+(z-a)^2=b^2$, a and b being constants are governed by
   (a) $xz_y-yz_x=0$, (b) $xz_y+yz_x=0$, (c) $yz_y-xz_x=0$,
   (d) $yz_y+xz_x=0$
7. The equation $\frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 z}{\partial y^2}$ is: (a) parabolic (b) Elliptic (c) Hyperbolic (d) None of these
8. The solution of the PDE $r-t=0$ is:
   (a) $z=f(x^2+y^2)$ (b) $f_1(y+x)+f_2(y-x)$
   (c) $f_1(y+x)+f_2(y-2x)$ (d) none of these
9. The Complete integral of $z=px+qy+c\sqrt{1+p^2+q^2}$ is__________
10. The PI for the PDE $(D^2+DD'+D'-1)z=4\cosh x$ is__________

Section B

Attempt any FIVE: **(5x6=30)**

1. Solve the linear partial differential equation $x^2\frac{\partial^2 z}{\partial x^2}-4xy\frac{\partial^2 z}{\partial xy}+4y^2\frac{\partial^2 z}{\partial y^2}+6y\frac{\partial z}{\partial y}=x^3y^4$
2. Solve $(D-3D'-2)^2z=2e^{2x}\tan(y+3x)$
3. Find the equation of the integral surface of the partial differential equation $2y(z-3)p+(2x-z)q=y(2x-3)$ which passes through the circle $x^2+y^2-2x=0, z=0$.
4. Solve the partial differential equation $u_{xx}=u_y+2y, u(0,y)=0, u_x(0,y)=1+e^{-3y}$ *by the method of seperation of variables.*
5. (i) Solve : $(y^2+z^2-x^2)p-2xyq+2xz=0$
   (ii) Solve: $(D^2-DD')z=\cos 2y(\sin x+\cos x)$
6. Solve the partial differential equation $x^2p^2+y^2q^2=z^2$

OR

If u is a function of $x, y,$ and z which satisfies $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$, show that u *contains* x,y,z only in combination of $(x+y+z)$ and $(x^2+y^2+z^2)$.

Section C

Attempt any FIVE: **(5x10=50)**

1. Reduced the equation $(n-1)^2\left(\frac{\partial^2 z}{\partial x^2}\right)-y^{2n}\left(\frac{\partial^2 z}{\partial y^2}\right)=ny^{2n-1}\frac{\partial z}{\partial y}$ to canonical form, and find the general solution.
2. A thin rectangular plate whose surface is impervious to heat flow has at $t=0$ an arbitrary distribution of temperature $f(x,y)$. Its four edges $x=0$, $x=a, y=0, y=b$ are kept at zero temperature. Determine the temperature at a point of the plate as t decreases. Also show a suitable diagram of this.

OR

A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially released rest from this position $y=y_0\sin^3\frac{\pi x}{l}$. If it is released rest from this position, find the displacement $y(x,t)$.

3. A surface is drawn satisfying $(D^2 + D'^2)z = 0$ touching $x^2 + y^2 = 1$along its section by $z = 0$. Prove that the required surface is $y^2(x^2 + y^2 - 1) = z^2(x^2 + y^2)$.

4. (a)Solve $\frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3} + \frac{\partial^3 u}{\partial z^3} - \frac{\partial^3 u}{\partial x\, \partial y \partial z} = x^3 + y^3 + z^3 - 3xyz$.

   (b)Solve the PDE $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$.

5. (a) Solve $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$.

   (b)Solve $(x^2 + xy)p - (xy + y^2)q + (x - y)(2x + 2y + z) = 0$.

6. (a)Find the general solution of the equation $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{3x+4y}\sin(x + 2y) + xy$.

   (b)Solve $\left(D^2 - D'^2\right)z = tan^3x\, tany - tanx\, tan^3y$.

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