12/12/16

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Third Semester, 2016

School of Physical Sciences

Integrated M.Sc.(Mathematics)

Course: MAC-106: Group Theory

Time Allowed: 3 Hours | Maximum Marks: 100

Note:

1. Attempt All Questions from Sections A.
2. Attempt Any seven Questions from Sections B.
3. Attempt Any three Questions from Sections C.

SECTION: A

($13 \times 1 = 13$ Marks)

1. The number of generator of a finite cyclic group of order 8 is .....
2. According to Fermat's theorem: If p is prime and a is any integer, then .....
3. The product of an odd permutation and an even permutation is an ..... permutation.
4. If $H = \{I, (1,2)\}$ and $K = \{I, (13)\}$ are two subgroups of a group G = S_3, then HK is ..... of G = S_3.
5. A group G of order 2p, where p is prime and $p > 2$ has exactly ..... subgroups of order p.
6. A subgroup N of a group G is said to be a normal subgroup of G if ...... for each $g \in G$ and $n \in N$.
7. Let G and G' be two groups under multiplication. If $f : G \to G'$ be a homomorphism, then f is one-one iff ....
8. The number of generators of an infinite cyclic group is ....
9. A finite cyclic group of order n is isomorphic to ....
10. If a group G has no non-trivial subgroups, then G must be finite group of .... order.
11. If H is a subgroup of G and N is a normal subgroup of G then what about the $H \cap N$ and H?
12. If a mapping $f : (C, +) \to (R, +)$ defined by $f(x + iy) = x$ is a homomorphism.Then the kernel of f is .....
13. A permutation $\sigma = (123)(45)(16789)(15)$ is ..... permutation.

SECTION: B

(7 × 6 = 42 Marks)

1. Show that the centre of a group is a normal subgroup of that group.
2. Define kernel of a homomorphism and show that if $f : G \to G'$ is a homomorphism then kernel of f is a normal subgroup of G.
3. If H and K are two subgroups of a group G, show that $H \cup K$ is a subgroup of group of G iff either $H \subset K$ or $K \subset H$.
4. If Z is the centre of a group G such that $\frac{G}{Z}$ is cyclic, then show that G is abelian.
5. If G is a group of order 35.Show that it cannot have two subgroups of order 7.
6. if A_3 be the subgroup of group S_3 consisting of all even permutations show that A_3 is normal subgroup of S_3, and order of A_3 is half of the order of S_3.
7. Prove that any two right cosets of H in G are either identical or disjoint, H being a subgroup of G.
8. If $G =< a >$ be a finite cyclic group of order n, then show that a^m is a generator of G iff $0 < m < n$ and $(m, n) = 1$.

SECTION: C

(15 × 3 = 45 Marks)

1. Define Quotient groups and show that if G is abelian group and N is normal subgroup of G then $\frac{G}{N}$ is abelian, however the converse need not be true.
2. State and prove Cayley's theorem.
3. State and prove of the Fundamental theorem of homomorphism.
4. Show that the set A_n of all even permutations of S_n is a normal subgroup of S_n and $o(A_n) = \frac{n!}{2}$.